IDEAS EN CIENCIAS DE LA INGENIERÍA

Reabastecimiento Conjunto para la Gestión de Inventarios de Partes de Repuesto: Caso de Estudio

Joint Replenishment for Spare Parts Inventory Management: Case Study

Sánchez Guzmán, V.[1], Loza-Hernandez, L.[1]*, Bustos Rosales, A.[2], Vázquez Aranda, S. [1]
(*llozahernandez@yahoo.com)

Recibido: 04-09-2025 Aceptado: 21-09-2025

Resumen

En el sector automotriz, específicamente en la rama de autopartes, equilibrar los costos y la disponibilidad de productos es una tarea compleja. El objetivo de este trabajo es proponer una política de inventario que minimice los costos totales de inventario y satisfaga la demanda de los clientes de una empresa de autopartes en México, además de mostrar soluciones alternativas a los problemas derivados de la falta de una gestión adecuada de inventarios. Para lograr este objetivo, se propuso una metodología para facilitar la puesta en práctica de los tomadores de decisiones, se realizó un análisis de las autopartes para seleccionar las de mayor interés para la empresa y pronosticar su demanda mediante modelos ARIMA. Posteriormente, se compararon las políticas de inventario para la revisión periódica y continua de dichos productos. Finalmente, se aplicó el sistema de reabastecimiento conjunto propuesto por Atkins e Iyogun a la mejor política que permite la existencia de faltantes y la ausencia de faltantes, logrando que los artículos con baja demanda se incluyan en los pedidos de artículos con alta rotación. Los resultados indican que la mejor política de inventario a aplicar en la compañía es la política de revisión continua.

Palabras clave: Gestión de inventarios, Política de reabastecimiento conjunto de autopartes, Inventario de revisión periódica, Inventario de revisión continua.

Abstract

In the automotive sector, specifically in the auto parts branch, balancing costs and product availability are an arduous task. The objective of this work is to propose an inventory policy that minimizes total inventory costs and satisfies customers' demand for an automotive spare parts company in Mexico, in addition to showing alternative solutions to problems derived from the lack of proper inventory

[1] Facultad de Ingeniería de la Universidad Autónoma del Estado de México. Paseo Universidad S/N. Toluca, Estado de México, México. CP. 501301.
[2] Instituto Mexicano del Transporte, Instituto Mexicano del Transporte, km 12+000 Carretera Estatal No. 431, El Colorado-Galindo Sanfandila, Pedro Escobedo, Querétaro, C.P. 76703 México 3.

management. To carry out the objective, it proposes a methodology to facilitate the implementation of decision makers, an analysis of the spare parts was performed to select those of greatest interest to the company and forecast their demand using ARIMA models, then inventory policies for periodic review and continuous review of those products are compared. Finally, the joint resupply system proposed by Atkins and Iyogun was applied to the best policy allowing shortages and no shortages, achieving that articles with low demand are included in the orders of articles with high turnover. The results indicate that the best inventory policy to apply in the company is the continuous revision policy.

Keywords: Inventory management, Joint replenishment for spare parts, Periodic review inventory policy, Continuous review inventory policy.

INTRODUCTION

Inventory management and control represent one of the most important activities in any organization [1]. The control of costs and the determination of efficient policies for maintenance and inventory management implies a reduction in expenses derived from its logistics or commercial activity and an increase in its profits [2].

Inventory management aims to determine and control inventory levels within physical inventory systems, so that the need for product availability and the need to minimize inventory-related costs are well balanced [3]. Another objective is to provide customers with a high level of service to avoid lost sales due to insufficient inventory [2]. According to Axsäter [4], four fundamental components determine the inventory systems: demands, replenishments, costs, and constraints.

The goal of inventory management is to establish an optimal inventory policy that controls stock replenishment, minimizes inventory-related costs, and maintains customer service levels. In short, an inventory policy determines when to place an order for replenishment and how much each product orders.

The company under study is Mexican and is dedicated to selling automotive spare parts for suspensions and steering systems nationwide. It distributes around 1,000 products to the national market. The company is considered a leader in the spare parts industry, offering products of equivalent quality to original parts. The company does not have an efficient inventory management tool that considers costs and controls the number of spare parts in stock.

Based on information provided by the company staff, future demand was determined (previously predicted using ARIMA (Autoregressive Integrated Moving Average) models [5], afterwards the appropriate inventory policies were calculated, so that the total cost of inventory management is minimized. Once the best inventory policy is found, a joint replenishment system proposed by Atkins and Iyogun [6] is used, allowing items with low demand to be included with fast moving item orders. Thus, it eliminates the need for higher configuration costs when the slow-moving item reaches its reorder

point. This work focuses on managing inventory with unknown or probabilistic demand and applying policies according to this type of demand. The selected policies are policy (s, S) (s-reorder point, S-target inventory) and the policy (R, S) (R-review period, S-target inventory). The goal is to determine the optimal policy for the inventory system so that the total cost across the entire system is minimized while meeting the target service level for customers for products that the company personnel determine as priorities.

This document shows the application of ARIMA forecasting models, as a basis to identify the demand for the products with fast movement of the company, case study; also, it presents in detail the inventory policy´s comparison of continuous review versus periodic review to define the supply of products jointly and to propose an inventory policy for these products, according to the needs of the company.

The document has four more sections: the literature review, the methodology, the case study and results obtained from the application of the new inventory policy, and finally conclusions.

LITERATURE REVIEW

The main objective of inventory management is to answer the questions: 1) How often should the inventory be reviewed? 2) When should it be replenished? and 3) What quantity should it be replenished to? [7]. The first question is answered according to the inventory control policy that companies apply to their inventories. The second question can be answered in two ways: i) inventory must be replenished when the quantity of product (stock) is equal to or less than s (reorder point) units. ii) inventory can be replenished every ***R*** (review period) unit of time. While the possible answers to the third question are: i) ***Q*** (fixed order quantity) units are always added to the inventory, or ii) a quantity is replenished such that, when it is incorporated into the inventory, the inventory level reaches the value of ***S*** (target inventory) units [5 - 7]. The main goal of these questions is to provide the desired level of customer service at a minimal cost. The first model to answer the third question was developed ten decades ago by Harris in 1913, who called this model EOQ (Economic Order Quantity) [8].

Inventory models can be classified into deterministic and stochastic models. Deterministic models assume a known demand and delivery time with certainty, on the other hand, stochastic models, demand, and delivery times are not known with certainty [7]. The objective of the probabilistic model is to determine the moment of issuance of the replenishment and the quantity to order, subject to the uncertainty of the demand and / or any other parameter [9].

Taking the above into account, the following inventory policies are considered: i) continuous review policies ***(s, Q)*** and ***(s, S)***. The policy ***(s, Q)*** requests a fixed quantity of **Q** each time the inventory position falls to the order point or lower, on the other hand, the policy ***(s, S)*** occurs when the stock decreases as it is ordered to raise the position of the inventory up to level ***S***. ii) periodic review policies ***(R, S)*** and ***(R, s, S)***, the policy ***(R, S)*** every ***R*** units of time, is ordered enough to raise the position of the inventory at level ***S*** [7]. Another variable is the delivery period (denoted by ***L*** known as lead time), which represents

the time interval that elapses between when an order is requested until the moment when the said order is added to stock [10 -15].

Due to the random component that the demands for spare parts have, it is necessary to show revision systems with probabilistic demand, allowing the safety stock calculus ***(S)*** which dampens the demand for spare parts [16]. For this case study, the continuous review policies ***(s, S)*** and the periodic review policy ***(R, S)***, proposed by [7] for inventory control with probabilistic demands, were selected, which are detailed below.

Policy (s, S)
This inventory control system assumes continuous review, and as the ***(s, Q)*** system is performed each time the inventory position falls to order point ***s*** or less. However, in contrast to the ***(s, Q)*** system, a variable replenishment quantity is used, ordering just enough to bring the inventory position up to level ***S*** [2 - 4, 9]. If all demand transactions are of unit size, both systems are identical because the replenishment request will always be made when the inventory position is exactly s; that is, ***S = s + Q*** [2]. This policy was addressed by [17], using mixed integer nonlinear programming to determine the almost optimal parameters and, later, using mixed integer linear programming for comparison and improving the model. For the application of the inventory management policy of continuous review ***(s, S),*** the formulas Ec. (1), Ec. (2), Ec. (3), Ec. (4), Ec. (5) y Ec. (6) proposed by [14] are used:

Order size:

$$Q = EOQ = \sqrt{\frac{2AD}{vr}} \quad (1)$$

where:
$A = cost\ \ of\ \ configuration\ to\ order$
$D = annual\ demand$
$v = product\ cost$
$r = maintenance\ rate$
Standard deviation of predicted errors on replacement time ***L***:

$$\sigma_L = \hat{\sigma}_1\sqrt{L} \quad (2)$$

where:
$\widehat{\sigma_1} = forecast\ standard\ deviation$
$L = lead\ time$
Special function of normal distribution:

$$G_u(k) = \frac{Q}{\sigma_L}(1 - P_2) \quad (3)$$

where:
$Q =$ *Order size*
$\sigma_L = \ Standard\ deviation\ of\ predicted\ errors\ on\ replacement\ time\ \boldsymbol{L}$
$P_2 = probability\ or\ rate\ of\ existence\ of\ the\ product$
Expected demand over replacement time ***L:***

$$s = \hat{x}_L + k\sigma_L \quad (4)$$

where:
$d = average\ monthly\ demand$
Reorder point:

$$s = \hat{x}_L + k\sigma_L \quad (5)$$

where:
$\hat{x}_L = Expected\ demand\ over\ replacement\ time\ \boldsymbol{L}$
$\sigma_L = Standard\ deviation\ of\ predicted\ errors\ on\ replacement\ time\ \boldsymbol{L}$
$k = security\ factor$
Relevant total cost:

$$TRC = \frac{AD}{Q} + \left[\frac{Q}{2} + k\sigma_L\right] vr + \frac{D}{Q}(B_2 v)\sigma_L G_u(k) \quad (6)$$

where:
$B_2 = fractional\ charge\ per\ missing\ unit$

Policy (R, S)

This inventory control system assumes a periodic review, it is also known as a replenishment cycle system, which is of common use particularly in companies without sophisticated computer control. It is also frequently seen, when items are ordered from the same vendor or require resource sharing. The control procedure is that each ***R*** time units (which is at each review instant) is ordered enough to raise the inventory position to level ***S***. The system ***(R, S)*** offers a regular opportunity (each ***R*** time units) to adjust the order down to the ***S*** level, a desirable point if the demand pattern changes over time. The main disadvantages of the system ***(R, S)*** are that replenishment quantities vary, and maintenance costs are higher than in continuous review systems [3].

For the application of the periodical review inventory management policy ***(R, S),*** the following formulas proposed by [3] are used Ec. (7), Ec. (8), Ec. (9) y Ec. (10):

Expected demand on replacement time ***L***:

$$\hat{x}_{R+L} = d(R + L) \quad (7)$$

where:
$R = review\ period$
Reorder point:

$$s = \hat{x}_{R+L} + k\sigma_{R+L} \quad (8)$$

where:

$$\sigma_{R+L} = \sqrt{\frac{R+L}{2}} * \sigma_1\sqrt{R} \quad (9)$$

Relevant total cost:

$$TRC = \frac{A}{R} + \left[\frac{DR}{2} + k\sigma_{R+L}\right] vr + \frac{1}{R}(B_2 v)\sigma_{R+L} G_u(k) \quad (10)$$

Where the variables integrated into Ec.8, 9, and 10 have the same meaning already described in the policy ***(s, S).*** In general, a certain amount of product must be kept in inventory for the efficient operation of a business. This quantity is conditioned or depends on the products' demand in the market. The demand must be satisfied with the amount of inventory stored. The classic inventory management models consider that the demand is known and constant. However, this situation, in which future demands are precisely known, is rare in practice [4, 16].

These two inventory policies, proposed in this case study, have been widely studied for the management of different types of inventories, a bibliographic review was carried out applying different methods for 2012 to 2015 for perishable items [17]. Studies for different types of demands have been developed, using mixed integer nonlinear programming, heuristics such as Silver-Meal and the Wagner-Whitin algorithm, which are promising for certain types of demands [20]. [21] proposed a Langrangian relaxation approach for capacity limitations to obtain optimal solutions in the planning of replenishment of multi-products with dynamic demand, which integrates customer returns to the inventory for future sales or their elimination if these spare parts reduce the capacity of the warehouse. [22] proposed an inventory system for automotive spare parts considering various levels according to backorders and discounts for quantities requested from suppliers. [23] present a non-linear programming model for entire order cycles and determines the optimal quantity of orders that propose the best inventory policy that maximizes the profits per period of the company; inventory policy allows for shortages, such as backorders that are satisfied with immediate inventory replenishment. [24], through numerical approximation algorithms, they seek to determine the optimal order cycle and inventory levels, such a way that the expected profits for the case study company are maximized. [25] suggest a heuristic to determine the suitable inventory of various products in accordance with an inventory ***policy (s-1, s)*** and the newspaper vendor type, which minimizes the expected value of the work in progress.

A joint replenishment system is when several items are ordered simultaneously rather than individually, the intention is to minimize the total cost by adding more line items to a replenishment. This is much less expensive than ordering items individually when the items are supplied by the same supplier [26]. A joint supply system looks for a balance between inventory review frequency decisions and order quantities; achieving a coordination of all products (items), considering that orders are made to a single supplier [8] or determine the supply policy that determines the joint order of products (independent demand) to minimize the cost of maintaining a production system, such as it is described by [27]. Within the review systems to perform joint replenishment, one of them is a periodic review system suggested by Atkins and Iyogun in 1988 [6], who developed an algorithm to perform a joint replenishment, which is adapted to the needs of the company under study.

This type of system aims to generate savings in ordering costs by allowing control over the main groups of items that can be ordered at the same time. These items are consumed simultaneously but fall within completely different review periods. This model developed by [6] makes possible to find the necessary quantities and the appropriate review periods for the products [7]. The Atkins and Iyogun procedure essentially allocate the highest cost by small quantity replenishment to the products that are produced or purchased more frequently, keeping the expected time for the next replenishment of these products in balance [3].

For the development of this algorithm, the ***EOQ*** is initially calculated using the Ec. (11):

$$EOQ_i = \sqrt{\frac{2a_i D_i}{rv_i}} \tag{11}$$

where:

$a_i =$ *lower cost of adding a product i*

$D_i = \ annual\ demand\ for\ product\ i$
$v_i = unit\ cost\ of\ product\ i$

At the same time, the equivalent time supply is calculated for each product, using only the lowest setup cost a_i assuming one year as 50 weeks Ec. (12).

$$T_i = \frac{EOQ_i(50)}{D_i}\ weeks \tag{12}$$

With this, a supply time T_i is obtained for each item within a product family, then the product with the smallest time supply is chosen and labelled as product 1. A small portion $\propto_1$ of the highest cost $\boldsymbol{A}$, is assigned to this product. The idea is to eventually allocate all the major cost to various products in the family. If all the cost were spread over a single product, its ***EOQ*** could increase significantly, same as its supply time [3].

Assigning more and more $\propto_1$ to the first product increases its supply time, eventually to the point where it matches the supply time of the second most frequently purchased product. Now, allocate more of the main installation or replenishment costs to product 1, and some to product 2, keeping supply times in balance. When these two equal supply times increase to the time supply of the third most frequently purchased product, start assigning $\propto_3$ to this third product.

This process continues until the entire main configuration is assigned, that is $\sum_i \propto_i = 1$. Therefore, the total cost of the setup is accounted for each time a family is ordered [3]. The total cost for an item is expressed as $(\alpha_i A + a_i)$, it is easy to solve for a supply time given by Ec. (13):

$$\alpha_i = \frac{T_i^2 r_i v_i D_i}{2A} - \frac{a_i}{A} \tag{13}$$

where:
$A = cost\ \ of\ \ configuration\ to\ order$

METHODOLOGY

This document proposes the methodology shown in Figure 1 to facilitate follow-up in solving the case study problem and similar problems in different productive sectors, which consists of six stages for its execution.

An explanation in detail of each of the stages is described below.

1. Identification of the system: This corresponds to the periodicity or frequency which the inventories are carried out in the company, characteristics of the products in terms of supply movement, and customer demand.
2. ABC Classification: A selection of products is made according to the relevance of the products through the Pareto method, which shows the classification of products based on their importance for the company's sales and the number of products. 80% of sales are represented by 20% of stock products [20].
3. Determination of demand: Based on the analysis of demand patterns, it means the data provided by the case study company shows stationarity, seasonality, trend, and variability.

https://doi.org/10.36677/rici.v3i3.26587 Número Especial 2025. ISSN: 2992-7447 págs. 71-92

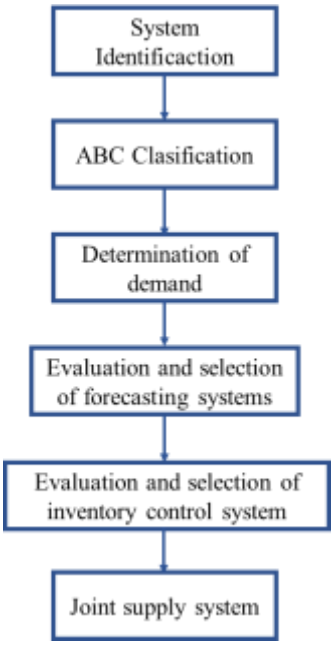


FIGURE 1: *Methodology proposed for inventory management. Source: Own elaboration.*

4. Evaluation and selection of forecasting systems: To calculate future demands based on the behavior of product demand. In this work classical forecasting models and ARIMA models were evaluated, where ARIMA models got the lowest forecast errors
5. Evaluation and selection of inventory control system: This work focuses on probabilistic inventory management systems where demand is uncertain. The applied policies are policy (s, S) and policy (R, S).
6. Joint supply system: At this point, a balance is sought between the frequency of inventory review decisions and the quantities to be ordered; achieving the coordination of all products, assuming that orders are made to a single supplier. The periodic review system applied is based on the algorithm suggested by [6].

However, this methodology can be applied to similar problems, where the goal is to develop an inventory policy considering specific characteristics of the products and productive sectors.

RESULTS

As it was mentioned, the company under study is Mexican-owned and was founded in 1982. It is a marketer of spare parts for suspension and steering in the auto parts market. It is renowned nationally as a supplier of automotive parts, handling around 1,000 SKUs (Stock Kee-ping Units). Currently the company does not have a spare parts inventory management and control tool, as expected, there are low service levels due to a lack of materials, as well as an excess of components and finished products in the warehouse, which justifies the development of this job.

The application of the proposed methodology for inventory management to the case study is as follows:

System identification

The company under study does not have a specific inventory control system, currently the inventory is controlled empirically based on the experience of the personnel in charge to the activity, which also, varies subjectively according to the experience of the staff, for this reason an adequate inventory management system is proposed for the selected products.

ABC classification

With the application of the Pareto principle, 160 articles of type A were obtained out of the 1,000 articles in total. The 15 items selected for this proposal belong to classification A, which are those that require major control due to their movement and customer demand (based on information provided by company staff).

Determination of demand

From the articles of classification A, it was concluded that they present demand with trend and, for some items their demand is stationary, generally showing variable demand. The 15 selected articles present trends, in addition to variability, and only one presents stationary demand. Figure 2, Figure 3, and Figure 4 show the demand behaviour of the products during the analyzed period.

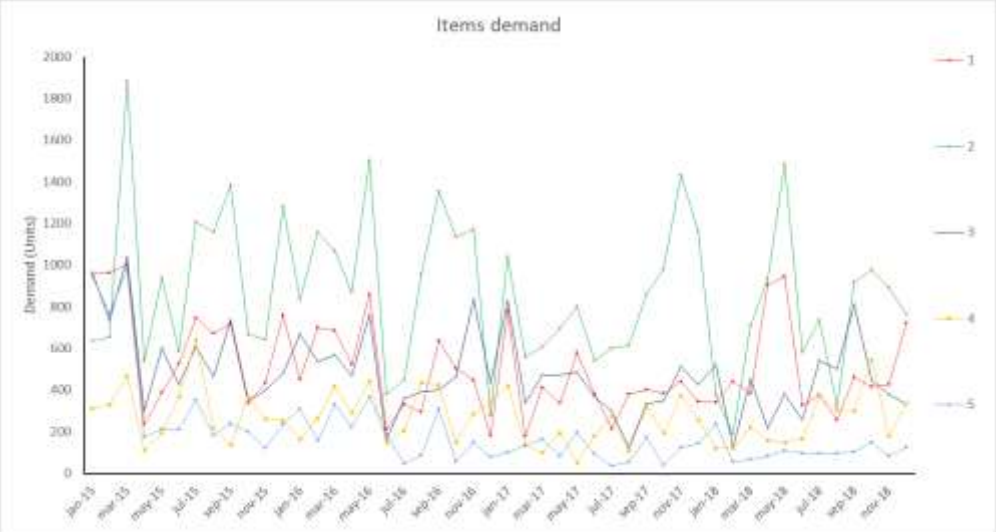


FIGURE 2: *Example of demand behaviour of products (1, 2, 3, 4, and 5) analyzed. Source: Own elaboration.*

https://doi.org/10.36677/rici.v3i3.26587 Número Especial 2025. ISSN: 2992-7447 págs. 71-92

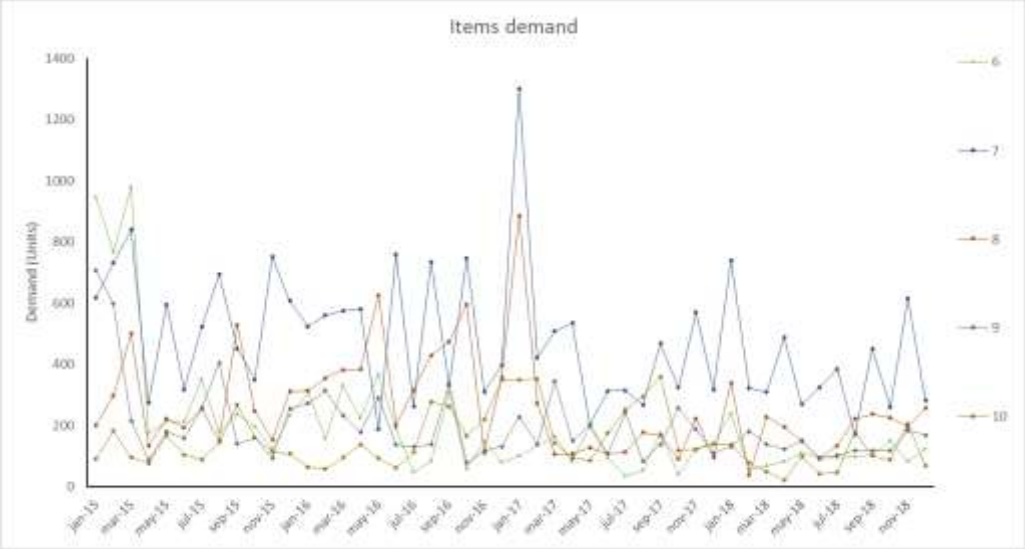


FIGURE 3: *Example of demand behaviour of products (6, 7, 8, 9, and 10) analyzed. Source: Own elaboration.*

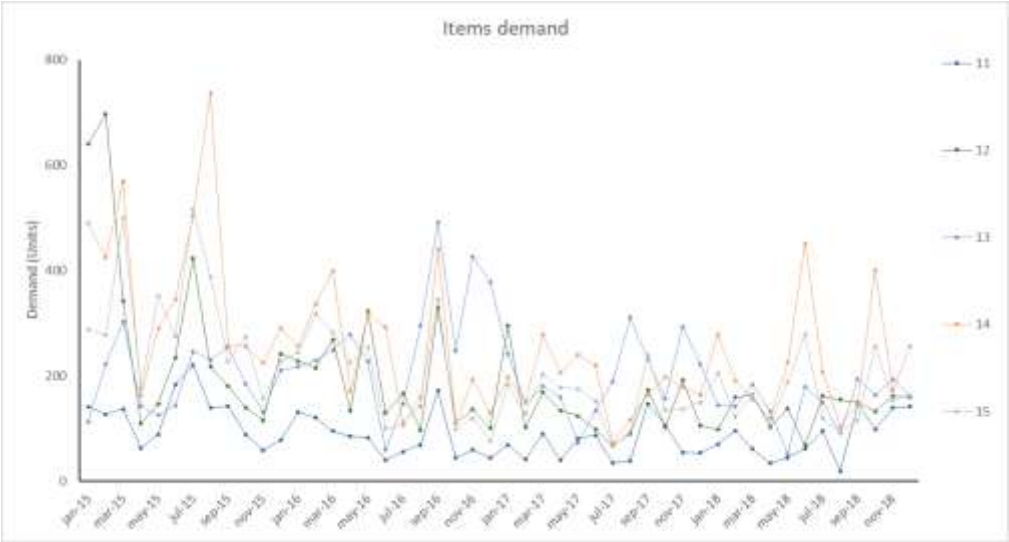


FIGURE 4: *Examples of demand behaviour of products (11, 12, 13, 14 and 15) analyzed. Source: Own elaboration.*

Figures 2, Figure 3, and Figure 4 show that the demand behavior of each of the products analyzed is very varied, to determine to calculate an adequate forecast for each of them.

Table 1 shows the measures of central tendency for each product and allows it to see the distribution of the demand, which shows that they have a very wide dispersion and do not behave normally as expected, therefore, they will need to be analyzed deeply to calculate the forecast for each product.

Table 1: Measures of central tendency of the demand.

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Mean	508	864	481	267	281	199	477	262	197	149	90	188	202	257	204
Standard Deviation	228	362	201	128	132	202	216	165	122	90	45	127	87	137	101
Variance	*51807*	*131342*	*40434*	*16364*	*17401*	*40644*	*46835*	*27244*	*14997*	*8108*	*2030*	*16088*	*7564*	*18719*	*10221*

Source: Own elaboration with information provided by company staff.

Evaluation and selection of forecasting systems

To determine the demand forecasts of the products, the classical quantitative forecasting methods were applied, however, the results obtained for the performance measures known as MAPE (mean absolute percentage error), MSD (mean square-red deviation) and MAD (mean of the absolute deviations) [5] were not satisfactory, so ARIMA models were applied, which presented the lowest mean absolute percentage error for selected products (an average of 15%) -[5] shows in detail the procedure to calculate forecasts of historical data series applying ARIMA method-. Subsequently, the demand forecasts for the 15 selected products were calculated (Table 2).

Table 2: Demand forecasts for selected products.

	Item														
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***
d (units/ month)	392	777	460	172	279	189	321	270	210	149	110	189	202	258	210

Source: Own elaboration with information provided by company staff

Evaluation and selection of the inventory control system

In this stage, a joint supply system is considered, which seeks a balance between the frequency of inventory review decisions and the quantities to be ordered, based on the data obtained from the characteristics of the products and the information got from the company staff. Then, a periodic review system suggested by [6] is applied.

In Table 3, the data for 15 products of the spare parts company are shown. The demand forecasts were previously obtained using ARIMA forecasting models, whose models yielded the lowest forecasting errors compared to classical forecasting methods, this demand data is used for the selection of the optimal inventory policy ***(s, S)*** or ***(R, S)***. Additional data is provided by the personnel of the company.

The Figure 5 shows the difference of safety stock between the inventory policies evaluated, where it is shown that there is a greater safety stock in the periodic review inventory policy, which directly impacts the total costs of the inventory policy.

Table 4 shows the comparison of the results obtained for the inventory policies proposed.

Table 3: Data for 15 selected products.

Constants	Items														
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***
d (units / month)	392	777	460	172	279	189	321	270	210	149	110	189	202	258	210
σ (units)	105	163	96	55	58	70	89	81	59	60	33	45	50	72	38
L (month)	3														
v ($ / units)	2.65	3.72	3.05	3.73	3.06	1.77	4.77	5.7	3.65	6.79	7.8	3.74	3.98	3.14	3.17
A ($ / order)	65														
r ($ / $ / year)	20%														
P_2	95%														
B_2	0.52	0.55	0.48	0.56	0.25	0.31	0.56	0.7	0.87	0.53	0.91	0.55	0.42	0.52	0.53
Backorders allowed															

Source: Own elaboration with information provided by company staff.

Table 4: Comparison of inventory policies.

Item	Inventory policy (units)		Safety Stock SS (units)	Relevant Total Cost TRC (DLS) Backorders	Difference (DLS)	Relevant Total Cost TRC (DLS) without backorders	Difference (DLS)
1	Continue **(s, S)**	(1676, 2933)	53	$ 916.54	$61.23	$ 933.60	$77.50
	Periodic **(R, S)**	(2, 2851)	134	$ 977.77		$ 1011.10	
2	Continue **(s, S)**	(2760, 4104)	144	$ 1989.61	$54.35	$ 2,033.51	$52.52
	Periodic **(R, S)**	(2, 4538)	190	$ 2043.96		$ 2,086.03	
3	Continue **(s, S)**	(1461, 2569)	29	$ 1,085.97	$47.62	$ 1,105.20	$47.04
	Periodic **(R, S)**	(2, 2501)	97	$ 1,133.59		$ 1,152.24	
4	Continue **(s, S)**	(822, 1923)	0	$ 384.00	$176.13	$ 387.28	$175.57
	Periodic **(R, S)**	(2, 1442)	77	$ 560.13		$ 562.85	
5	Continue **(s, S)**	(834, 1678)	0	$ 642.32	$83.00	$ 649.06	$79.83
	Periodic **(R, S)**	(2, 1454)	59	$ 725.32		$ 728.89	
6	Continue **(s, S)**	(535, 1135)	19	$ 676.47	$120.71	$ 686.39	$118.47
	Periodic **(R, S)**	(2, 949)	89	$ 797.18		$ 804.86	
7	Continue **(s, S)**	(993, 1593)	30	$ 1,270.36	$61.85	$ 1,285.67	$71.72
	Periodic **(R, S)**	(2,1749)	144	$ 1,332.21		$ 1,357.39	
8	Continue **(s, S)**	(837, 1437)	27	$ 1,409.69	$70.77	$ 1,440.52	$65.70
	Periodic **(R, S)**	(2, 1481)	131	$ 1,480.46		$ 1,506.22	
9	Continue **(s, S)**	(650, 1250)	20	$ 902.57	$101.52	$ 922.14	$101.08
	Periodic **(R, S)**	(2, 1145)	95	$ 1,004.09		$ 1,023.22	
10	Continue **(s, S)**	(467, 1067)	20	$ 954.12	$105.70	$ 968.23	$106.70
	Periodic **(R, S)**	(2, 842)	97	$ 1,059.82		$ 1,074.93	
11	Continue **(s, S)**	(341, 941)	11	$ 1,006.37	$99.67	$ 1,030.13	$97.12
	Periodic **(R, S)**	(2, 604)	54	$ 1,106.04		$ 1,127.25	
12	Continue **(s, S)**	(582, 1182)	15	$ 881.74	$96.70	$ 900.40	$96.77
	Periodic **(R, S)**	(2, 1018)	73	$ 978.44		$ 997.17	
13	Continue **(s, S)**	(623, 1223)	17	$ 710.33	$92.90	$ 720.30	$90.92
	Periodic **(R, S)**	(2, 1091)	81	$ 803.23		$ 811.22	
14	Continue **(s, S)**	(798, 1398)	24	$ 766.47	$99.99	$ 778.99	$98.75
	Periodic **(R, S)**	(2, 1406)	116	$ 866.46		$ 877.74	
15	Continue **(s, S)**	(643, 1243)	13	$ 667.47	$86.22	$ 678.62	$84.5
	Periodic **(R, S)**	(2, 1112)	62	$ 753.69		$ 763.21	

Source: Own elaboration with information provided by company staff



FIGURE 5: *Comparison of inventory policies through safety stock. Source: Own elaboration.*

The Figure 6 shows the difference between the inventory policies evaluated considering inventory shortages and without shortages, which shows that there is a greater difference in the total costs of the policies when there are shortages, directly impacting on the total cost and customer service, the latter being one of the most important indicators in the evaluation of inventory policies.

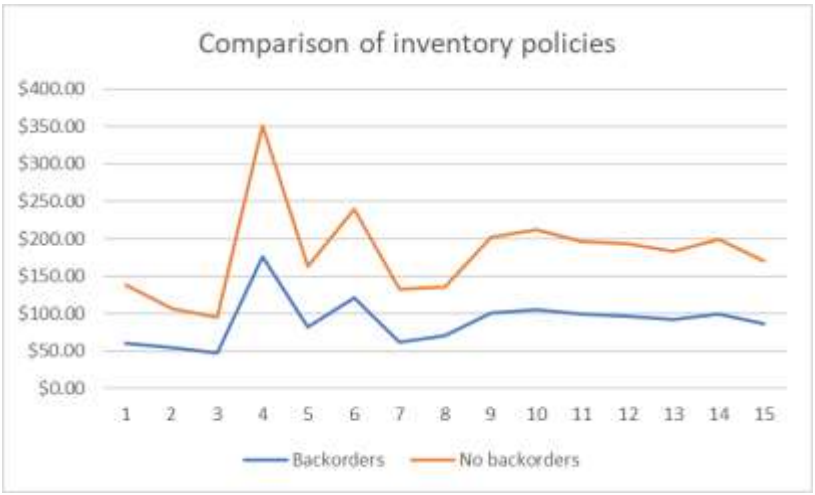


FIGURE 6: *Comparison of inventory policies with shortages and without shortages. Source: Own elaboration.*

In addition, for the policy of periodic review, the Atkins and Iyogun [6] algorithm for a joint replenishment is applied, the results are shown in Table 5, 6, and 7. These results clearly indicate that the best inventory policy to apply in the company is the continuous revision policy, since it would save $ 1,515.11 (DLS) per year for these 15 articles studied.

Joint supply system

Following the policy of continuing review and applying the Atkins and Iyogun algorithm for joint replenishment with backorders allowed, the results shown in Table 5 and 6 are obtained, also results for joint replenishment without backorders allowed are shown in Table 7:

Table 5: Calculations *EOQ* and *T*.

Item	Demand (units)	v (DLS)	*EOQ* (units)	*T* (weeks)
1	5964	2.65	300	2.52
2	11280	3.72	348	1.54
3	6672	3.05	296	2.22
4	2724	3.73	171	3.14
5	4044	3.06	230	2.84
6	3108	1.77	265	4.26
7	4920	4.77	203	2.06
8	4212	5.7	172	2.04
9	3228	3.65	188	2.91
10	2508	6.79	122	2.42
11	1716	7.8	94	2.73
12	2808	3.74	173	3.09
13	3024	3.98	174	2.88
14	3960	3.14	225	2.84
15	2976	3.17	194	3.26

Source: Own elaboration with information provided by company staff.

Table 6: Values obtained using Atkins and Iyogun.

Item	α_i	*T* (weeks)	Rounding *T*	*EOQ* (units)
1	0.066	3.276	4	478
2	0.216	3.276	4	903
3	0.091	3.276	4	534
4	0.026	3.276	4	218
5	0.063	3.276	4	324
6	0	3.276	4	249
7	0.121	3.276	4	394
8	0.139	3.276	4	337
9	0.041	3.276	4	259
10	0.07	3.276	4	201
11	0.04	3.276	4	138
12	0.012	3.276	4	225
13	0.024	3.276	4	242
14	0.064	3.276	4	317
15	0.027	3.276	4	239

Source: Own elaboration with information provided by company staff.

Following the Atkins and Iyogun algorithm, the α_i are calculated for each product, and the new periods and EOQ are obtained for each product with backorders allowed. Table 6 shows the results.

Table 6 shows the optimal order quantity to be requested in the rounded week period (column 4), the $\boldsymbol{\alpha}$ value indicates the proportional amount that is increased to $\boldsymbol{Q}$ to satisfy the demand in the number of

weeks in which the joint order will be made, for this policy shortages are allow at the end of the period, so it is necessary for all cases to multiply by the obtained alpha constant; which as indicated in the introduction requires the application of iterations to identify the best value.

These new review periods and EOQ are the ones that best suit the company to generate savings in costs for joint replenishment without backorders. Table 7 shows the results.

Table 7: Values obtained using Atkins and Iyogun algorithm.

Item	α_i	T (weeks)	Rounding T	EOQ (units)
1	-	7.45	6	716
2	0.7111	5.26	4	902
3	0.2889	6.45	6	801
4	-	9.45	8	436
5	-	8.41	8	647
6	-	4.26	4	249
7	-	6.45	6	590
8	-	5.26	4	337
9	-	9.04	8	516
10	-	7.45	6	301
11	-	8.41	8	275
12	-	9.04	8	449
13	-	8.63	8	484
14	-	8.63	8	634
15	-	9.45	8	476

Source: Own elaboration with information provided by company staff.

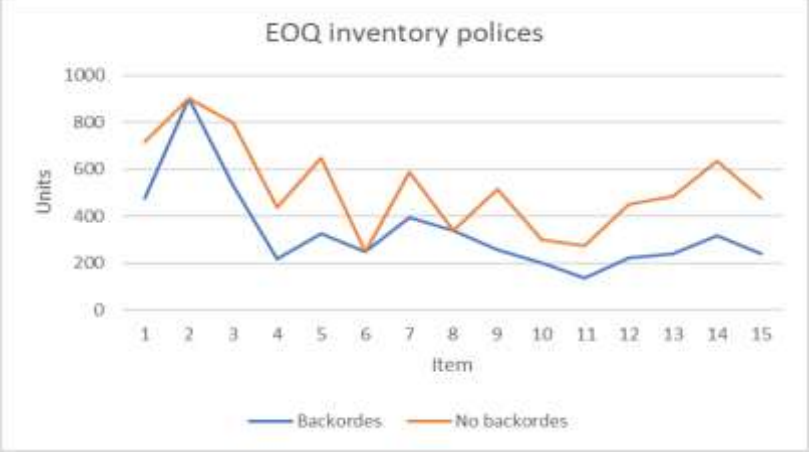


FIGURE 7: *Comparison of EOQ for the inventory policy with shortages and without shortages. Source: Own elaboration.*

Similarly, as detailed in Table 6, Table 7 shows the optimal order quantity that should be requested in the rounded week period to satisfy the demand in the number of weeks in which the joint order will be made; for this policy, shortages are not allowed at the end of the period. Figure 7 shows the differences.

Figure 6 shows the difference in optimal order quantities, which demonstrates that when shortages are allowed, the quantity ordered in period T is lower, while if shortages are not allowed, the optimal order quantity increases considerably, which is subsequently reflected in the total inventory cost.

CONCLUSIONS

The automotive parts industry is complex due to the variability of its products' demand and the importance of maintaining high standards, [28 - 30] shows an example of this situation in the auto parts industry. Forecasts are a powerful tool that copes with these types of difficulties and the expected changes in the future, thus contributing to the generation of economic benefits for companies.

Product demand forecasting enables the development of adequate inventory policies that minimize cost while considering the level of service offered by companies. The inventory models used are classified based on assumptions about demand, review periods, shortages, costs, and lead times. Additionally, it is important to mention that all models seek to optimize their replenishment policies by minimizing their total costs, which in most models a fixed order cost is used, a maintenance cost of inventory and missing costs.

Regarding the inventory policies used, it is recommended to prioritize probabilistic methods since in itself the demand for the products of this company always tends to be variable. Additionally, good product coordination reduces resupply costs, as observed in the case study of automotive spare parts companies. This allows items with low demand to be included in orders of high-demand items.

On the other hand, this article contributes to the literature in this area of knowledge by proposing a methodology that is intuitively and separately applied in the productive sector by people responsible for developing these activities. Furthermore, this work applies the methodology, which allows for determining an inventory policy considering the characteristics of the products and the detailed analysis of the demand for each of them to establish an inventory policy with joint replenishment. It should be noted that based on the reviewed literature, companies of all sizes and different sectors avoid using this type of inventory policy due to its complexity, even when savings in their total costs are demonstrable. Therefore, the application of the proposed methodology serves as support to decision-makers and personnel dedicated to these tasks in the different productive sectors.

STATEMENT OF CONTRIBUTION OF AUTHORS AND CONTRIBUTORS

Sánchez Guzmán, V.: Data curation, Research, Writing - Preparation of the original draft**. Loza-Hernandez, L.:** Conceptualization, Methodology, Research, Writing. **Bustos Rosales, A.:** Review and Editing. **Vázquez Aranda, S.:** Supervision and Validation.

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